Inference Rules

Both in classical and intuitionistic logics:

$$\frac{}{\Gamma;\ A;\ \Delta \vdash A}\ (\textit{Axiom}) \qquad \frac{\Gamma;\ A;\ A;\ \Delta \vdash C}{\Gamma;\ A;\ \Delta \vdash C}\ (\textit{Copy})$$

$$\frac{\Gamma;\ \Delta \vdash C}{\Gamma;\ A;\ \Delta \vdash C}\ (\textit{Weakening}) \qquad \frac{\Gamma;\ \Delta;\ \Gamma';\Delta' \vdash C}{\Gamma;\ \Gamma';\Delta;\ \Delta' \vdash C}\ (\textit{Swap})$$

$$\frac{}{\Gamma\ \vdash\ \top}\ (\textit{True-intro}) \qquad \frac{}{\Gamma;\ \bot;\ \Delta\ \vdash\ C}\ (\textit{False-elim})$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \vee B}\ (\textit{Or-intro-1}) \qquad \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B}\ (\textit{Or-intro-2})$$

$$\frac{\Gamma;\ A;\ \Delta \vdash C \qquad \Gamma;\ B;\ \Delta \vdash C}{\Gamma;\ A \vee B;\ \Delta \vdash C}\ (\textit{Or-elim})$$

$$\frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \wedge B}\ (\textit{And-intro}) \qquad \frac{\Gamma;\ A;\ B;\ \Delta \vdash C}{\Gamma;\ A \wedge B;\ \Delta \vdash C}\ (\textit{And-elim})$$

$$\frac{\Gamma;\ A \vdash B}{\Gamma \vdash A \Rightarrow B}\ (\textit{Imp-intro}) \qquad \frac{\Gamma;\ \Delta \vdash A \qquad \Gamma;\ B;\ \Delta \vdash C}{\Gamma;\ A \Rightarrow B;\ \Delta \vdash C}\ (\textit{Imp-elim})$$

$$\frac{\Gamma;\ A \vdash \bot}{\Gamma \vdash \neg A}\ (\textit{Not-intro}) \qquad \frac{\Gamma;\ \Delta \vdash A}{\Gamma;\ \neg A;\ \Delta \vdash C}\ (\textit{Not-elim})$$

Only in classical logic:

$$\frac{\Gamma;\ \neg A \vdash \bot}{\Gamma \vdash A}\ (\textit{Proof by contradiction})$$