Some Proofs from Lecture 1

1. $\neg A$ and $A \Rightarrow \bot$ are equvalent

$$\cfrac{\cfrac{\cfrac{}{A \vdash A}\;(\textit{Axiom})}{\neg A;\ A \vdash \bot}\;(\textit{Not-elim})}{\neg A \vdash A \Rightarrow \bot}\;(\textit{Imp-intro})$$

Note: when we applied the (*Imp-intro*), we had rule's A = our A, rule's B = our $\bot$ and empty Γ. When we applied (*Not-elim*), we had rule's A = our A, rules's C = our A, empty Γ and $\Delta = A$.

$$\cfrac{\cfrac{\cfrac{}{A \vdash A}\;(\textit{Axiom}) \qquad \cfrac{}{\bot;\ A \vdash \bot}\;(\textit{Axiom})\ \text{or}\ (\textit{False-elim})}{A \Rightarrow \bot;\ A \vdash \bot}\;(\textit{Imp-elim})}{A \Rightarrow \bot \vdash \neg A}\;(\textit{Not-intro})$$

2. Principle of excluded middle

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{}{A \vdash A}\;(\textit{Axiom})}{A \vdash A \vee \neg A}\;(\textit{Or-intro-1})}{\neg(A \vee \neg A);\ A \vdash \bot}\;(\textit{Not-elim})}{\neg(A \vee \neg A) \vdash \neg A}\;(\textit{Not-intro})}{\neg(A \vee \neg A) \vdash A \vee \neg A}\;(\textit{Or-intro-2})}{\neg(A \vee \neg A);\ \neg(A \vee \neg A) \vdash \bot}\;(\textit{Not-elim})}{\neg(A \vee \neg A) \vdash \bot}\;(\textit{Copy})}{\vdash A \vee \neg A}\;(\textit{Proof by contradiction})$$