

CS101 (Fall 2005) Special Topics in Computer Science
Language-Based Security

Homework 2, Part I, Problem 2b

$$\begin{array}{c}
 \frac{\frac{\lambda x.(\lambda y.(x (y x))) \longrightarrow \lambda x.(\lambda y.(x (y x)))}{\lambda x.(\lambda y.(x (y x)))} \quad \frac{\lambda x.(x+1) \longrightarrow \lambda x.(x+1)}{\lambda x.(x+1)} \quad \frac{\lambda y.(\lambda x.(x+1) (y \lambda x.(x+1))) \longrightarrow \lambda y.(\lambda x.(x+1) (y \lambda x.(x+1)))}{\lambda y.(\lambda x.(x+1) (y \lambda x.(x+1)))}}{\lambda x.(\lambda y.(x(yx)) \lambda x.(x+1) \longrightarrow \lambda y.(\lambda x.(x+1) (y \lambda x.(x+1)))} \quad \frac{\lambda x.(x \ 1) \longrightarrow \lambda x.(x \ 1)}{\lambda x.(x \ 1)} \quad \frac{\frac{\lambda x.(x+1) \longrightarrow \lambda x.(x+1)}{\lambda x.(x \ 1) \lambda x.(x+1) \longrightarrow 2} \quad \frac{\dots}{\lambda x.(x \ 1) \lambda x.(x+1) \longrightarrow 2} \quad \frac{\frac{2 \longrightarrow 2 \quad 1 \longrightarrow 1}{2+1 \longrightarrow 3}}{3 = 2 + 1}}{\lambda x.(x+1) (\lambda x.(x \ 1) \lambda x.(x+1)) \longrightarrow 3} \\
 \hline
 \lambda x.(\lambda y.(x (y x)) \lambda x.(x+1) \lambda x.(x \ 1) \longrightarrow 3
 \end{array}$$

$$\begin{array}{c}
 \frac{\frac{\lambda x.(x \ 1) \longrightarrow \lambda x.(x \ 1)}{\lambda x.(x \ 1)} \quad \frac{\lambda x.(x+1) \longrightarrow \lambda x.(x+1)}{\lambda x.(x+1)} \quad \frac{\frac{1 \longrightarrow 1 \quad 1 \longrightarrow 1}{1+1 \longrightarrow 2}}{2 = 1 + 1}}{\lambda x.(x+1) \lambda x.(x+1) \longrightarrow 2} \quad \frac{\lambda x.(x+1) \longrightarrow \lambda x.(x+1)}{(\lambda x.(x+1)) \ 1 \longrightarrow 2}}{\lambda x.(x \ 1) \lambda x.(x+1) \longrightarrow 2}
 \end{array}$$