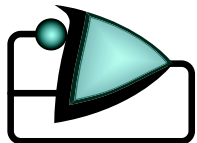


CS101C

Type Theory and Formal Methods

Lecture 9

April 30, 2003



Rules

$$\frac{}{\Gamma; x: A; \Delta \vdash x \in A} \text{ (type_axiom)} \qquad \frac{\Gamma \vdash t_2 \in C \quad \Gamma; x: C \vdash t_1[x] \in T}{\Gamma \vdash t_1[t_2] \in T} \text{ (cut)}$$

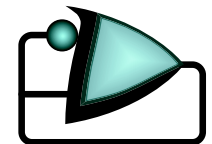
$$\frac{}{\Gamma; x: \text{Void}; \Delta \vdash C[x]} \text{ (void-E)} \qquad \frac{}{\Gamma \vdash \cdot \in \text{Unit}} \text{ (unit-T)}$$

$$\frac{\Gamma; x: A \vdash t[x] \in B}{\Gamma \vdash (\lambda x. t[x]) \in (A \rightarrow B)} \text{ (\lambda-T)} \qquad \frac{\Gamma; x: (A \rightarrow B); \Delta \vdash t_1[x] \in A \quad \Gamma; y: B; \Delta \vdash t_2[y] \in C}{\Gamma; x: (A \rightarrow B); \Delta \vdash t_2[x t_1[x]] \in C} \text{ (\rightarrow-E)}$$

$$\frac{\Gamma \vdash a \in A \quad \Gamma \vdash b \in B}{\Gamma \vdash (a, b) \in (A \times B)} \text{ (pair-T)} \qquad \frac{\Gamma; u: A; v: B; \Delta \vdash t[(u, v)] \in C}{\Gamma; x: (A \times B); \Delta \vdash t[x] \in C} \text{ (pair-E)}$$

$$\frac{\Gamma \vdash a \in A}{\Gamma \vdash \text{inl}(a) \in (A + B)} \text{ (inl-T)} \qquad \frac{\Gamma \vdash b \in B}{\Gamma \vdash \text{inr}(b) \in (A + B)} \text{ (inr-T)}$$

$$\frac{\Gamma; u: A; \Delta[\text{inl}(u)] \vdash t[\text{inl}(u)] \in C[\text{inl}(u)] \quad \Gamma; v: B; \Delta[\text{inr}(v)] \vdash t[\text{inr}(v)] \in C[\text{inr}(v)]}{\Gamma; x: A + B; \Delta[x] \vdash t[x] \in C[x]} \text{ (union-E)}$$



Rules

$$\frac{}{\Gamma; A; \Delta \vdash A} \text{ (type_axiom)} \qquad \frac{\Gamma \vdash C \quad \Gamma; C \vdash T}{\Gamma \vdash T} \text{ (cut)}$$

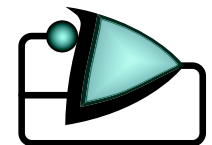
$$\frac{}{\Gamma; \text{Void}; \Delta \vdash C[x]} \text{ (void-E)} \qquad \frac{}{\Gamma \vdash \text{Unit}} \text{ (unit-T)}$$

$$\frac{\Gamma; A \vdash B}{\Gamma \vdash (A \rightarrow B)} \text{ (\lambda-T)} \qquad \frac{\Gamma; (A \rightarrow B); \Delta \vdash A \quad \Gamma; B; \Delta \vdash C}{\Gamma; (A \rightarrow B); \Delta \vdash C} \text{ (\rightarrow-E)}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash (A \times B)} \text{ (pair-T)} \qquad \frac{\Gamma; A; B; \Delta \vdash C}{\Gamma; (A \times B); \Delta \vdash C} \text{ (pair-E)}$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash (A + B)} \text{ (inl-T)} \qquad \frac{\Gamma \vdash B}{\Gamma \vdash (A + B)} \text{ (inr-T)}$$

$$\frac{\Gamma; A; \Delta \vdash C \quad \Gamma; B; \Delta \vdash C}{\Gamma; A + B; \Delta \vdash C} \text{ (union-E)}$$



Rules

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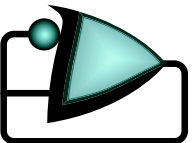
$$\frac{}{\Gamma; \text{Void}; \Delta \vdash C[x]} \text{ (void-E)} \quad \frac{}{\Gamma \vdash \text{Unit}} \text{ (unit-T)}$$

$$\frac{\Gamma; A \vdash B}{\Gamma \vdash (A \rightarrow B)} \text{ (\lambda-T)} \quad \frac{\Gamma; (A \rightarrow B); \Delta \vdash A \quad \Gamma; B; \Delta \vdash C}{\Gamma; (A \rightarrow B); \Delta \vdash C} \text{ (\rightarrow-E)}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash (A \times B)} \text{ (pair-T)} \quad \frac{\Gamma; A; B; \Delta \vdash C}{\Gamma; (A \times B); \Delta \vdash C} \text{ (pair-E)}$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash (A + B)} \text{ (inl-T)} \quad \frac{\Gamma \vdash B}{\Gamma \vdash (A + B)} \text{ (inr-T)}$$

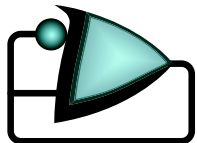
$$\frac{\Gamma; A; \Delta \vdash C \quad \Gamma; B; \Delta \vdash C}{\Gamma; A + B; \Delta \vdash C} \text{ (union-E)}$$



Propositions-as-Types

For every intuitionistic proposition, we construct a type of all the possible evidence for that proposition. The proposition is true when the type is non-empty.

Proposition	Type
\perp	Void
\top	Unit
$A \Rightarrow B$	$A \rightarrow B$
$A \wedge B$	$A \times B$
$A \vee B$	$A + B$



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???	$x : A \rightarrow B[x]$
???	$x : A \times B[x]$

